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A Study on Multi-Criteria Decision-Making (MCDM) Methods and its Applications

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Abstract – Multi-criteria decision-making (MCDM) encompasses diverse methods and methods designed to facilitate decision-making in situations where multiple, often conflicting criteria must be evaluated. MCDM techniques aim to provide a structured and systematic framework to prioritize alternatives, allocate resources, and make informed choices that balance different criteria. Elimination and Choice Expressing Reality (ELECTRE), Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), and Multi-Attribute Utility Theory (MAUT) are MCDM methods. The main goal of this review is to provide a concise overview of the fundamental principles and methodologies of MCDM. By leveraging these methods, decision-makers can systematically address the complexity and multi-dimensionality of real-world problems, ensuring more balanced and rational outcomes. The continuous development and integration of MCDM approaches highlight their significance in advancing decision science and enhancing decision quality in various domains.

Keywords: MCDM, TOPSIS, AHP, MAUT, ELECTRE, PROMETHEE.

1. Introduction

Multiple conflicting criteria decision problems can be solved methodologically with the help of MCDM. When evaluating and ranking alternatives, MCDM considers numerous factors at once, in contrast to traditional decision-making methods that concentrate on a single criterion (such as profit maximization or cost reduction). It allows the involvement of various values and domain types while it also ensuring that these entities do not make an explicit translation into a common domain [3]. The process is applied for a wide range of problems, from car or laptop selection to selecting technical policies. In MCDM functioning, complex problems are often broken down into minor components. These components are adjudged and after consideration, these are rearranged to solve the issue.

Characteristics of MCDM Methods:

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1. **Multiple Criteria**: MCDM deals with decision problems where there are two or more conflicting criteria that need to be considered simultaneously. These criteria could be qualitative or quantitative.
2. **Preference Modelling**: MCDM methods typically involve eliciting preferences from decision-makers or stakeholders regarding the relative importance of criteria and their preferences for different alternatives.
3. **Trade-offs**: MCDM acknowledges that different alternatives may perform differently across various criteria, and decision-makers need to make trade-offs among these criteria to identify the best compromise solution [1].

An MCDM can be expressed as a two-dimensional matrix, which is called a performance matrix. In short, MCDM consists of the following key components:

- Alternatives that need to be compared or chosen from
- The standards used to assess and compare the alternatives
- Weights indicating how important each criterion is relative to the others
- The decision-makers who will reflect their preferences [2]

Table 1Performance matrix

		Criterion			
		C_1	C_2	…..	C_n
Alternatives	a_1	v_{11}	v_{12}	…..	v_{1n}
	a_2	v_{21}	v_{22}	…..	v_{2n}
	.	.	.	.	.
		.	.	.	.
	a_m	v_{m1}		…..	v_{mn}

The relationship between the criteria for decision-making and the available choices is described in the performance matrix. The group of substitutions,

A= {A1, A2, A3…. An} forms rows, and

Set of Criteria,

C= {C1, C2, C3,.., Cn} forms columns in the performance matrix.

Each cell, V_{ij} denotes the decision maker’s preference for alternative i concerning criterion j.

2. Classification of MCDM Methods

MCDM methods can be classified into following two major classes: Compensatory and Non-Compensatory [4].

2.1 Compensatory Methods

These methods allow for trade-offs among criteria. A slightly unfavourable score is acceptable if it is compensated by one or more criteria with a highlyfavourable score. These methods are divided into four classes. Scoring methods, Compromising methods, Concordance methods, and Evident Reasoning methods.

2.2 Non- Compensatory methods

These methods do not permit trade-offs between criteria. An unfavorable score in one criterion cannot be compensated by a favorable score of other criteria. Thus, comparisons are made on a criteria-to-criteria basis. These methods are simple to understand and use. Examples of these methods include the dominance method, the Conjunctive constraint method, the Maxman Method, and the Disjunctive constraint method.Despite this, MCDM encompasses a wide range of techniques. The most widely used techniques for choosing, evaluating, and contrasting different options are, nonetheless, TOPSIS, ANP, ELECTRE, AHP, and PROMETHEE [11].

3. Types of Non-compensatory MCDM Approaches

There are several types of MCDM methods, each with its approach to handling multiple criteria and alternatives. Some common methods include:

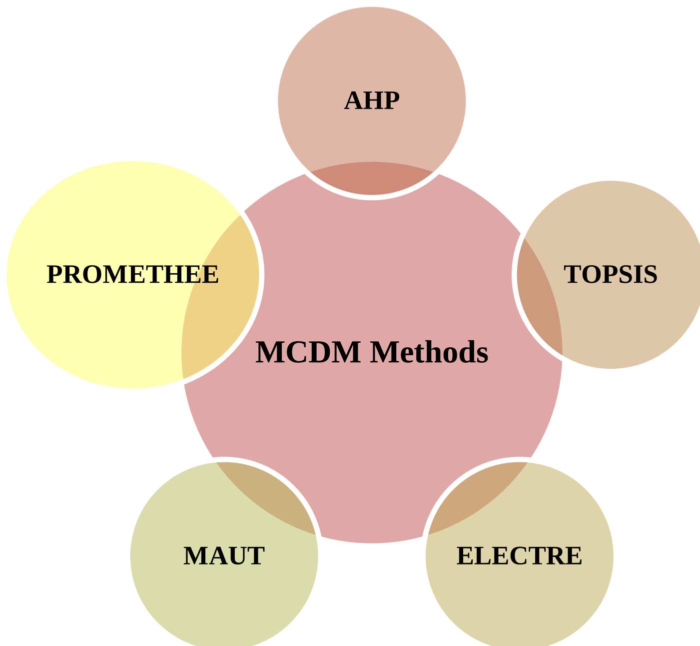

Figure 1 Types of Non-compensatory MCDM Approaches

3.1 Analytical Hierarchy Process (AHP)

To facilitate pairwise comparisons and the aggregation of priorities, the AHP is a structured technique that organizes and analyses complex decisions into a hierarchical framework of criteria and alternatives. Since its creation by Thomas L. Saaty in the 1970s, it has found extensive application across a range of disciplines, including social sciences, engineering, and business. The following lists the elements and procedures that make up the AHP:

Components of AHP

1. **Hierarchical Structure**: A decision question is broken down using AHP into a hierarchical framework of criteria and options.At the top level is the main objective or goal, followed by criteria that contribute to achieving the goal, and then alternatives which are evaluated against those criteria.
2. **Pairwise Comparisons:** A key component of AHP is pairwise comparison, in which decision-makers assess each pair of components of a certain criterion at a given level of the hierarchy. Typically, a numerical scale reflecting the relative importance or preference of one factor over another is used for these comparisons.
3. **Consistency Check:** To guarantee the validity of the pairwise comparisons, AHP incorporates a consistency check. To determine how inconsistent the decision-maker's comparisons are, the consistency ratio is computed. The comparisons need to be adjusted if the ratio rises above a particular threshold.
4. **Priority Vectors**: AHP computes priority vectors for every level of the hierarchy based on pairwise comparisons. The relative weights or priority of the items inside each level are represented by these vectors.

5. **Aggregation**: Priority vectors are aggregated through a process called synthesis to determine overall priorities or rankings of alternatives at the higher levels of the hierarchy.

Steps in AHP

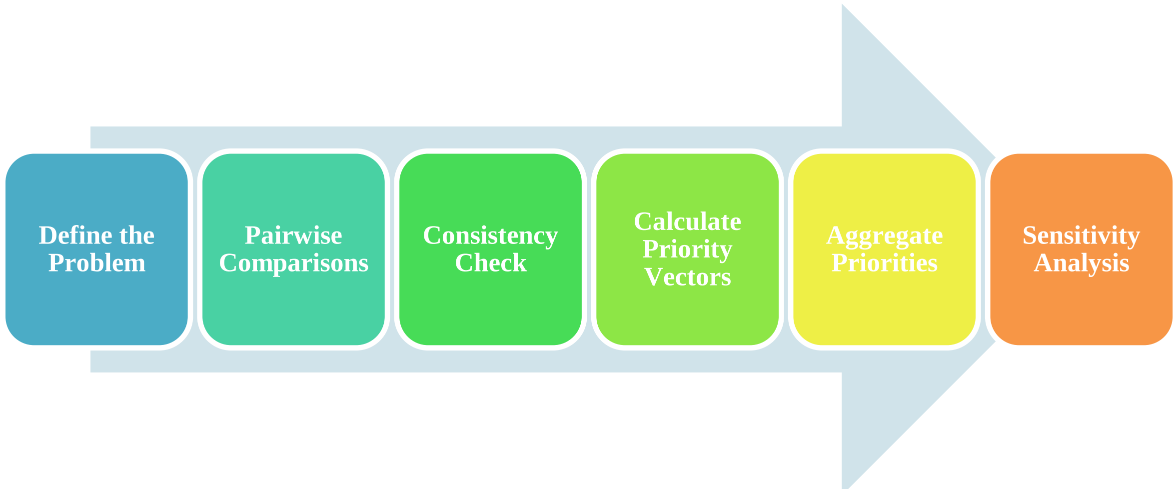

Figure 2 AHP Steps

1. **Define the Problem**: Clearly state the issue that needs to be resolved and create a hierarchical framework with objectives, standards, and possible solutions.
2. **Pairwise Comparisons:** In each level of the hierarchy, evaluate each pair of components concerning the level above in terms of relevance or preference.
3. **Consistency Check:** Use the consistency ratio to assess how consistent pairwise comparisons are. If necessary, modify the comparisons to get a satisfactory degree of consistency.
4. **Determine Priority Vectors:** Using pairwise comparisons as a basis, determine the priority vectors for each tier of the hierarchy.
5. **Aggregate Priorities:** To ascertain the overall priorities of the options, aggregate the priority vectors from the criteria level to the goal level.
6. **Sensitivity Analysis**: Use sensitivity analysis to evaluate how resilient the findings are to modifications in the pairwise comparisons [7].

3.2 Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

To rank alternatives, TOPSIS takes to account many variables, including the shortest path to the optimal solution (best performance) and the furthest path to the worst solution. TOPSIS, which was created by Hwang and Yoon in 1981, which looks at several factors to determine the optimal option by calculating the farthest distance to the worst solution as well as the shortest distance to the perfect answer.

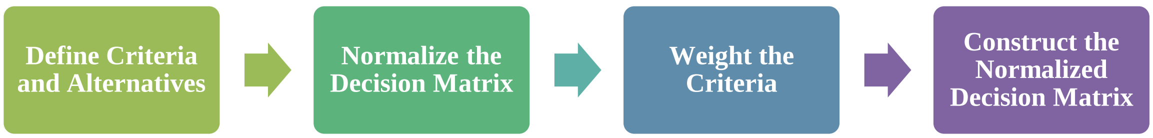

Figure 3 Steps in TOPSIS

1. **Define Criteria and Alternatives**:
   - Determine the standards (qualities) by which the options will be considered.
   - List all available alternatives that are being considered for the decision.
2. **Normalize the Decision Matrix**:
   - Create a decision matrix in which the criteria are represented by columns and the choices by rows.
   - Normalize the matrix to ensure that the values are consistent across various criteria. This stage guarantees a fair comparison of criteria with varying scales or units.
3. **Weight the Criteria**:
   - Give each criterion a weight to represent its relative importance or priority.
   - There are several ways to estimate the weights, including using expert opinion, analytical approaches like AHP, or calculating how important each criterion is to the choice.
4. **Construct the Normalized Decision Matrix**:
   - Divide each element in the decision matrix by the square root of the sum of squares of all the entries in the corresponding column to normalize the matrix [9].

3.3 ELimination Et Choix Traduisant la REalité (ELECTRE)

The term refers to a collection of multi-criteria decision-making (MCDM) techniques created by Bernard Roy in the 1960s and is translated as Elimination and Choice Expressing Reality. It is widely used to rank and select alternatives based on multiple criteria. The ELECTRE method is particularly useful when decision-makers want to handle qualitative criteria and consider the preferences of stakeholders.ELECTRE offers a methodical and organized way to make decisions based on a variety of factors, emphasizing transparency and consideration of stakeholder preferences. It is particularly suited for scenarios where qualitative aspects and thresholds play a significant role in decision outcomes.

Key Concepts of ELECTRE

1. **Preference Relation and Concordance**: ELECTRE assesses the concordance and discordance between alternatives based on pairwise comparisons of criteria. Concordance measures how well an alternative meets or exceeds a certain threshold for each criterion, while discordance measures the degree to which one alternative is worse than another for at least one criterion.
2. **Thresholds and Dominance**: ELECTRE uses thresholds to define acceptable performance levels for each criterion. Thisalgorithm also evaluates dominance relationships between alternatives based on these thresholds and pairwise comparisons.
3. **Classification**: After evaluating concordance and discordance, ELECTRE classifies alternatives such as "preferred," "indifferent," and "non-preferred." This classification helps decision-makers identify which alternatives are most suitable based on the defined criteria.
4. **Robustness Analysis**: Sensitivity analysis is possible using ELECTRE to evaluate how resilient rankings are to modifications in thresholds or weights for the criterion. This aids in decision-makers' comprehension of how stable their choices are in various situations.

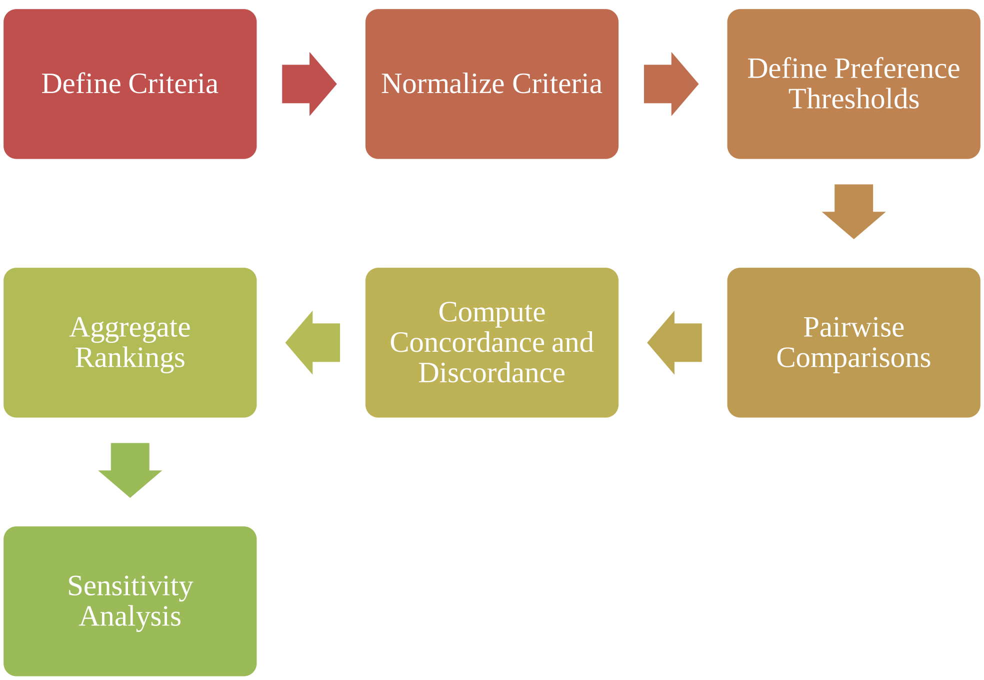

Figure 4 Steps in the ELECTRE Method

1. **Define Criteria:** Determine and specify the standards by which the alternatives will be judged. Both qualitative and quantitative criteria may be used.
2. **Normalize Criteria:** To make sure the criteria are on the same scale, normalize them.This step is crucial for combining different types of criteria into a unified evaluation framework.

3. **Define Preference Thresholds**: Set thresholds for each criterion to specify acceptable levels of performance. These thresholds reflect the minimum acceptable performance for an alternative to be considered viable.
4. **Pairwise Comparisons**: Compare each pair of alternatives with respect to each criterion to determine their relative performance. This involves assessing whether one alternative is preferable to another based on the established thresholds.
5. **Compute Concordance and Discordance**: Calculate concordance indices to measure the degree to which each alternative meets or exceeds the thresholds. Similarly, compute discordance indices to quantify the extent to which one alternative is worse than another for at least one criterion.
6. **Aggregate Rankings**: Aggregate the results of concordance and discordance to rank alternatives. Classify alternatives (preferred, indifferent, non-preferred) based on their overall performance relative to others.
7. **Sensitivity Analysis**: Use sensitivity analysis to assess how modifications to thresholds or weights for criteria impact how alternatives are ranked and categorized [10].

3.4 Multi-Attribute Utility Theory (MAUT)

It is a framework for making decisions that makes it easier to rank and evaluate options according to a variety of factors or characteristics.It combines decision-maker preferences with utility functions to assess and compare different alternatives in a structured manner. By combining qualitative and quantitative elements into a single assessment framework, MAUT offers a methodical and structured approach to decision-making in the face of uncertainty and complexity. It helps decision-makers to make well-informed decisions that complement their priorities and preferences across a range of factors.

Components of Multi-Attribute Utility Theory (MAUT):

1. **Attributes or Criteria**: The first step in MAUT is to determine the pertinent characteristics or standards that will be applied to assess the options. These criteria can be qualitative or quantitative and should capture the essential dimensions of the decision problem.
2. **Utility Functions**: Decision-makers assign utility functions to each criterion to quantify their preferences or satisfaction levels. Utility functions map the performance of alternatives on each criterion to a numerical scale representing the decision-makers perceived value or utility.
3. **Weighting of Criteria**: Decision-makers can assign a weight to each criterion based on its relative importance by using MAUT. These weights represent the decision-maker's choices and priorities among the several criteria. The weighting process can be done using various techniques, such as direct elicitation from stakeholders or through analytical methods like AHP (Analytical Hierarchy Process).
4. **Scoring or Evaluation of Alternatives**: Each alternative is evaluated against the criteria using the utility functions and weights assigned. This step involves calculating a utility score for each alternative, which aggregates the contributions of all criteria based on their respective weights and utility values.

5. **Aggregation of Utility Scores**: Once utility scores are computed for all alternatives, MAUT aggregates these scores to rank the alternatives. Based on the decision-makers preferences, the option with the highest total utility score is deemed the best or most favoured option.
6. **Sensitivity Analysis**: Sensitivity analysis using MAUT can be used to investigate how modifications to utility functions or criteria weights impact the ranking of alternatives. This study aids in determining the decision's resilience and elucidates the effects of uncertainty or preference fluctuations [6].

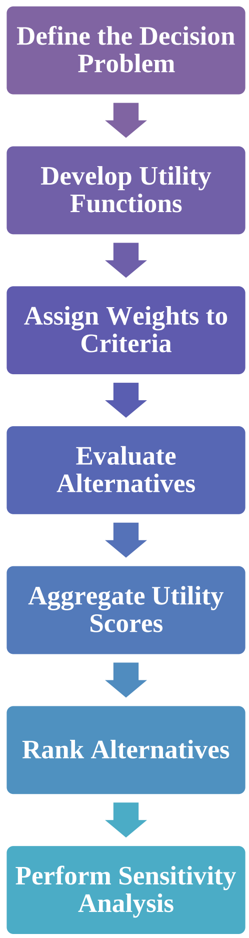

Figure 5 Steps in MAUT

1. **Define the Decision Problem**: Clearly define the objectives and criteria for decision-making, ensuring all relevant aspects are captured.
2. **Develop Utility Functions**: Create utility functions that measure the decision-maker's preferences for each criterion. These functions can be linear, nonlinear, or even based on qualitative assessments.
3. **Assign Weights to Criteria**: Based on the relative importance of each criterion to the decision-maker, assign weights to them. Ensure weights reflect the trade-offs and priorities among criteria.

4. **Evaluate Alternatives**: Evaluate each alternative against the criteria using the utility functions and weights assigned.
5. **Aggregate Utility Scores**: Aggregate the utility scores for each alternative to determine their overall utility or satisfaction level.
6. **Rank Alternatives**: Sort options according to the total utility scores assigned to them. Usually, the option with the highest score is the one that is chosen.
7. **Perform Sensitivity Analysis**: Sort options according to the total utility scores assigned to them. Usually, the option with the highest score is the one that is chosen [2].

3.5 Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE)

It is a branch of the MCDM algorithm designed to rank options according to their preference profiles. Developed by Brans and Vincke in the early 1980s, PROMETHEE methods are widely used in various fields to support decision-making processes that involve multiple conflicting criteria. PROMETHEE methodologies offer a flexible and straightforward approach to multi-criteria decision-making, accommodating both qualitative and quantitative criteria while considering decision-makers preferences in a structured manner. They are particularly useful when decision problems involve complex interactions among multiple criteria and alternatives.

Key Concepts of PROMETHEE

1. **Preference Function**: PROMETHEE methods use preference functions to assess the degree of preference that decision-makers have for one alternative over another concerning each criterion. These preference functions can be linear, Gaussian, or other forms that reflect the decision-maker's preferences.
2. **Preference Index**: For each pair of alternatives, PROMETHEE calculates a preference index that quantifies the net outranking flow from one alternative to another. The preference index is computed based on the difference between the positive outranking flow (for which the first alternative is preferred to the second) and the negative outranking flow (for which the second alternative is preferred to the first).
3. **Net Flow**: The net outranking flow is aggregated across all pairs of alternatives to determine a net preference index for each alternative. This index represents the overall degree to which an alternative is preferred over all others based on the specified criteria.
4. **Ranking**: Based on the net preference indices, alternatives are ranked in descending order. The alternative with the highest net preference index is considered the most preferred, while the one with the lowest index is considered the least preferred.
5. **Sensitivity Analysis**: Sensitivity analysis using PROMETHEE techniques is possible to evaluate how resilient rankings are to modifications in preference functions or criteria weights. This aids in decision-makers' comprehension of how stable their choices are in various situations.

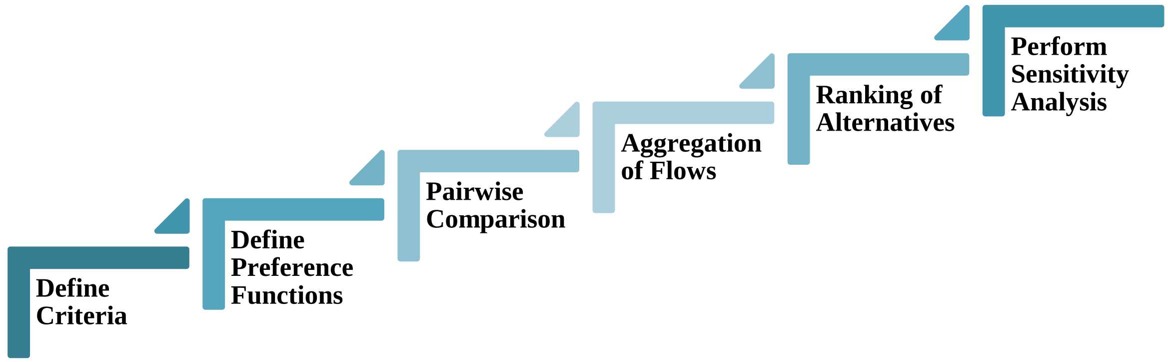

Figure 6 Steps in PROMETHEE

1. **Define Criteria**: Determine and specify the standards by which the alternatives will be judged. Both qualitative and quantitative criteria may be used.
2. **Define Preference Functions**: Specify preference functions for each criterion to reflect the decision-maker's preferences. These functions describe how the performance of alternatives on each criterion influences their rankings relative to others.
3. **Pairwise Comparison**: Compare each pair of alternatives for each criterion using the defined preference functions. Calculate the outranking flows (positive and negative) between alternatives for each criterion.
4. **Aggregation of Flows**: Aggregate the outranking flows to compute the net outranking flow for each alternative.
5. **Ranking of Alternatives**: Sort the options according to their net preference indices. First is the option with the highest index, and so forth.
6. **Perform Sensitivity Analysis**: To determine how altering preference functions or criterion weights may affect the rankings, conduct sensitivity analysis [10].

4. APPLICATIONS OF MCDM

MCDM methods are used in ranking of alternatives in various domains.

- **Environmental Management**: Evaluating and selecting environmental projects based on criteria like environmental impact, cost-effectiveness, and social acceptance.
- **Healthcare Decision Making**: Ranking medical treatments based on criteria such as efficacy, side effects, cost, and patient preferences.
- **Financial Investment**: Selecting investment portfolios considering criteria like risk, return, liquidity, and ethical considerations.

- **Transportation Planning**: Choosing transportation infrastructure projects based on criteria such as environmental impact, cost, and social equity.
- **Supplier Selection**: Evaluating and ranking potential suppliers based on criteria such as quality, delivery time, cost, and reliability [9].
- **Decision Making**: MCDM is used for decision-making in complex situations where multiple criteria and alternatives exist.
- **Resource Allocation**: It helps in prioritizing resource allocation based on criteria such as cost, time, and effectiveness.
- **Project Selection**: Choosing among alternative projects based on criteria such as feasibility,cost, and environmental impact.
- **Product Design**: It assists in product design by evaluating different design alternatives against customer needs and technical criteria.
- **Location Selection**: Determining the optimal location for a new facility based on criteria such as proximity to markets, transportation infrastructure, and labor availability.
- **Product Design**: Evaluating different design alternatives based on criteria like performance, cost, and customer satisfaction.
- **Policy Decision Making**: Selecting policies or interventions based on criteria such as effectiveness, feasibility, and social impact.
- **Urban Planning**: Selecting development projects or infrastructure investments based on criteria like sustainability, cost-effectiveness,and social impact [8].

5. Conclusion

The effectiveness of MCDM methods lies in their ability to integrate quantitative and qualitative data, accommodate stakeholder preferences, and provide transparent, repeatable decision processes. The continuous evolution and refinement of MCDM methodologies underscore their critical role in decision science. As decision-making challenges grow more intricate with technological advancements and increasing data availability, the relevance and application of MCDM methods will only expand. Embracing these approaches can lead to improved decision quality, greater stakeholder satisfaction, and more achievement of organizational and societal goals.

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Biography

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